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Optimal Commodity Taxes Under Rationing

Nanak Kakwani
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The results on a standard optimal commodity model change substantially when one or more commodities are rationed. The authors propose a more realistic model of rationing that overcomes some restrictive features of earlier rationing models.

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How useful and relevant are the results of standard optimal commodity tax models when one or more commodities are rationed? Kakwani and Ray investigated the implications of optimal commodity taxation under rationing and reached these conclusions:

- In a single-person economy, optimal policy dictates that the rationed commodity bears the entire tax. The implication for developing countries using this model is that if the government has a fixed budget to subsidize certain commodities, optimal policy will be to subsidize only the rationed commodities, such as food.

- In a many-person economy (which reflects reality), optimal policy will tax all nonrationed commodities at an infinite rate if the rule is that taxes on all commodities are proportional to prices.

- The widely used linear expenditure system cannot be used to find a sensible optimal commodity tax structure under rationing.

- The more a society is concerned about inequality, the greater the tax should be on nonrationed commodities.

Kakwani and Ray present an alternative (more realistic) model of rationing that overcomes some of the restrictive features of the previous rationing model.

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Optimal Commodity Taxes in the Presence of Rationing

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OPTIMAL COMMODITY TAXES IN THE PRESENCE OF RATIONING

1. INTRODUCTION

Commodity taxes play an important role in resource mobilisation in many developing countries [see Tanzi (1987)]. The predominance of indirect taxes in the overall tax effort in such economies reflects, partly, the severe political constraints that exist on direct taxes. The authorities in many less developed economies have traditionally, and more so in recent years, also relied heavily on indirect taxes as the principal tool for securing redistribution [see, for example, the recent volume edited by Newbery and Stern (1987)]. The commodity tax rate is, hence, a parameter of considerable policy importance in a developing country.

The literature on optimal commodity taxes dates back to Ramsey's (1927) seminal contribution which sought to answer the question (p.47): 'if a given revenue is to be raised by proportionate taxes on some or all uses of income... being possibly at different rates, how should these rates be adjusted in order that the decrement of utility may be a minimum? Ramsey's original treatment was devoted exclusively to the efficiency issue and ignored the equity/redistributive aspects of commodity taxation. Diamond and Mirrlees (1971) introduced optimal taxation in a many person economy and used a social welfare function approach which allowed examination of the equity aspect of indirect taxation.

The 'efficiency' aspect of commodity taxation essentially involves the government raising a pre-specified amount of aggregate revenue in the 'least cost' way - i.e. the authorities choose a set of commodity tax rates that maximise social welfare, expressed as a function of individual welfare/utility. It is clear that the solution, namely, the optimal tax rates would depend on: (a) society's perception of individual welfare, and

(b) the assumed form for consumer preferences, namely, the adopted utility/demand functional forms. The importance of assumptions about consumer preferences for optimal commodity taxes is now widely accepted in the literature [see, for example, Atkinson and Stiglitz (1980), Ch. 12, 14)]. As Ray (1986) has, recently, shown using numerical demand estimates for India, optimal commodity taxes are extremely sensitive to departure from the linearity/separability assumptions of the Linear Expenditure System (LES) demand functional form that is widely used in tax studies [Atkinson and Stiglitz (1972), Harris and Mackinnon (1978), Deaton (1975), Ahmad and Stern (1984), Deaton and Stern (1986)].

Even setting aside the question of the wisdom of using LES demand estimates in view of the well known incompatibility of the linearity/separability assumptions with data [Deaton (1974), Ray (1985a)], there is the issue of the relevance of optimal commodity tax theory to developing countries because of its dependence on the common assumption underlying empirical demand analysis, namely, that given his/her budget and commodity prices, the consumer has chosen freely the quantities that he/she is observed to have purchased. It is a matter of common observation that the assumption of free choice is unlikely to be valid in reality. There are various reasons why the quantities consumed may not be directly under the control of those who consume them. The most obvious example is that of necessities like Food, which are scarce in relation to demand in developing countries, and are subject to rationing. Since optimal commodity tax estimates are crucially dependent on the calculated price/expenditure elasticities, and since the latter are likely to be sensitive to the introduction of quantity constraint, the formulation of the standard Ramsey-Diamond-Mirrlees optimal tax problem in the presence of rationing is an important issue, especially in the case of LDCs. It is worth noting that while the optimal tax literature, especially in recent years, has concentrated on the link between optimal tax theory and consumer preferences, no attention has been paid to the question of the robustness of optimal commodity taxes to the presence of quantity constraint on demand viz. rationing. That is the central motivation of this paper.

Not surprisingly, some of the earliest studies on consumer demand under rationing took place during and soon after the War[1] [e.g. Rothbarth (1941), Tobin and Houthakker (1951)]. After a period of relative neglect in the sixties and early seventies, there has recently been renewed interest in the subject - e.g. Pollak [1969], Howard [1977], Weitzman [1977], Neary and Roberts [1980], Sah [1987]. The methodology of generating rationed demand equations rests on the concept of 'virtual prices' introduced by Rothbarth [1941] and developed recently in the elegant analysis of Neary and Roberts [1980] using duality and expenditure functions. The 'virtual price' of the rationed commodity is defined as that price which, in conjunction with actual prices of the non-rationed commodities and a 'virtual income' i.e. with the consumer compensated for the price change, will induce him to choose the rationed level as the result of (i.e. as if as) free choice. In their important contribution, Neary and Roberts [1980] have shown that virtual prices must exist, that the support or virtual prices of unrationed goods coincide with actual prices, and that, via the use of the expenditure function, one can generate a matching system of rationed demand equations from an unrationed demand system. It is, however, interesting to note that in most of the recent exercises on rationed demand systems (e.g. Neary and Roberts [1980], Sah [1987]), the LES has been chosen as an illustrative example. This is of course, not without reason, since LES is one of the few demand systems that

[1]See Tobin (1952) for a survey of these early studies.

generate an explicit expression for virtual price and, hence, the rationed LES is simple to write down and easy to estimate. A principal finding of this study, however, is that the rationed LES, in the context of optimal commodity tax theory, has some strong (and absurd) implications that should rule it out as a candidate for welfare applications.

In the context of the present exercise, only one commodity is rationed and the remaining are freely chosen items. It is important to note that, because of the 'complete systems' framework, quantity constraint on one item will have an impact on the demand levels and price/expenditure elasticities of the unrationed items, and hence on their optimal commodity taxes. Section 2 investigates the structure of optimal commodity taxes in a single person economy under rationing. The validity of the result, derived therein, in a many person economy is investigated in Section 3, using the most general Bergson-Samuelson form of social welfare function. In order to get some stronger results, an additive separable social welfare function of Atkinson's (1970) type is assumed in Section 4 with implications for optimal taxation under rationing outlined. The theory developed in the latter section is applied in Section 5 to the familiar case of the rationed LES, to derive some restrictive (indeed absurd) optimal tax results that cast serious doubt on the rationing framework as far as considered in this paper. Section 6 presents an alternative (and a more realistic) model of rationing where the rationed commodity can be bought up to the quantity of ration in the ration shops at a subsidised price, and beyond in the open market at the market price. As this section demonstrates, this more realistic rationing model has significant implications for optimal tax theory and helps to overcome some of the restrictive features of the previous rationing model. The paper ends on the concluding note of section 7.

2. SINGLE PERSON CASE

Suppose there are (n+1) commodities in the economy and one of them is rationed. Let rationed commodity be q_0 and denote the fixed quantity to be consumed by $\bar{q}_0$ and a price p_0 is charged for this commodity. Further, let q $(q_1, q_2, \ldots, q_n)$ be the vector of n freely chosen goods and $p = (p_1, p_2, \ldots, p_n)$ be the vector of prices. Then, the indirect utility function may be denoted by

$$u = u(\bar{q}_0, p_0, p, x)$$

where x is the lump-sum income of the household.

The optimal tax problem under rationing involves maximizing u subject to the government revenue constraint

$$R = \sum_{i=1}^{n} t_i q_i + t_0 q_0 \tag{2.1}$$

t_i being the tax rates. $(p_i - t_i)$ for $i = 0,1,2,\ldots,n$, are the producer prices which are assumed to be fixed in face of changes in the pattern of demand. Setting up the Lagrangean

$$L = u + \lambda R$$

and differentiating L with respect to p_0 and p_j and equating derivatives to zero, the first order conditions may be written as

$$\left[\frac{\partial u}{\partial p_0}\right]\left[\frac{\partial R}{\partial p_j}\right] = \left[\frac{\partial u}{\partial p_j}\right]\left[\frac{\partial R}{\partial p_0}\right] \tag{2.2}$$

where j varies from 1 to n.

Using Roy's theorem, it can be shown that

$$\frac{\partial u}{\partial p_0} = -\bar{q}_0 \lambda \quad \text{and} \quad \frac{\partial u}{\partial p_j} = -q_j \lambda \tag{2.3}$$

where $\lambda = \frac{\partial u}{\partial x}$ is the marginal utility of income.

Let us write the Slutsky equations

$$s_{i0} = \frac{\partial q_i}{\partial p_0} + \bar{q}_0 \frac{\partial q_i}{\partial x} \tag{2.4}$$

and

$$s_{ij} = \frac{\partial q_i}{\partial p_j} + q_j \frac{\partial q_i}{\partial x} \tag{2.5}$$

where s_{ij} is the compensated change in q_i with respect to p_j; i and j vary from 1 to n.

Since s_{ij} must be symmetric, (2.4) will imply $s_{i0} = 0$ for all i which means that there is zero substitution possibility into or away from the rationed commodity. Then differentiating (2.1) with respect to p_0 and p_j yields

$$\frac{\partial R}{\partial p_0} = \bar{q}_0 + \sum_{i=1}^{n} t_i \frac{\partial q_i}{\partial p_0} \tag{2.6}$$

and

$$\frac{\partial R}{\partial p_j} = q_j + \sum_{i=1}^{n} t_i \frac{\partial q_i}{\partial p_j} \tag{2.7}$$

respectively. Utilizing the fact that $s_{i0} = 0$ for all i and $s_{ij} = s_{ji}$, (2.6) and (2.7) can be written as

$$\frac{\partial R}{\partial p_0} = \bar{q}_0(1-\phi) \tag{2.8}$$

and

$$\frac{\partial R}{\partial p_j} = q_j(1+\delta_j-\phi) \tag{2.9}$$

respectively, where

$$\phi = \sum_{i=1}^{n} \alpha_i m_i, \tag{2.10}$$

$m_i = p_i \frac{\partial q_i}{\partial x}$ being the marginal propensity to spend, $\alpha_i = \frac{t_i}{p_i}$ and

$$\delta_j = \sum_{i=1}^{n} \alpha_i \eta^*_{ji}, \tag{2.11}$$

η^*_{ji} being the compensated price elasticities for non-rationed commodities.

Substituting (2.3), (2.8) and (2.9) into (2.2) leads to $\delta_j = 0$ for all j, so that

$$\sum_{i=1}^{n} \alpha_i \eta^*_{ji} = 0 \tag{2.12}$$

for j = 1, 2,...,n. The only solution to (2.12) occurs when $\alpha_i = 0$ for all i = 1,...,n; i.e. commodity tax rates on non-rationed goods are all equal to zero, so that rationed good bears the entire tax.

In the context of developing countries, this result has an important policy implication. If the government has a fixed budget to subsidize certain commodities, the optimal policy will be to subsidize only the rationed commodities (non-rationed commodities should receive zero subsidy).

It should be noted however, that this result is much less startling than it appears. The conventional wisdom in the optimal tax structure is that lump sum taxes are preferable to distortionary commodity taxes. In the present context, tax on the rationed items is like a lump sum tax, since there is zero substitution possibility into or away from the rationed item. It is hardly surprising, therefore, that the rationed item bears the entire tax. Does this result generalize to the many person case? The answer is provided in the next section.

3. MANY PERSON CASE UNDER GENERAL WELFARE FUNCTION

In a many person economy, the government is concerned with the maximization of a social welfare function. We assume that the social welfare function is of the Bergson-Samuelson form which is a function

$$W = W(u^1, u^2, \ldots, u^H)$$

of the utilities of the H individuals. The optimal taxation problem involves choosing t_i $(i = 0,1,\ldots,n)$ to maximize W subject to the revenue constraint

$$\bar{R} = \sum_{i=1}^{n} t_i Q_i + t_0 H \bar{q}_0 \qquad (3.1)$$

where $Q_i = \sum_{h=1}^{H} q_i^h$ being the aggregate demand of the ith commodity.

Using the Lagrangean framework, the first order optimization conditions may be

$$\left[\frac{\partial W}{\partial p_0}\right] \left[\frac{\partial \bar{R}}{\partial p_j}\right] = \left[\frac{\partial W}{\partial p_j}\right]\left[\frac{\partial \bar{R}}{\partial p_0}\right] \qquad (3.2)$$

where j varies from 1 to n.

Using Roy's identity

$$\left[\frac{\partial W}{\partial p_0}\right] = -\bar{q}_0 \sum_{h=1}^{H} \beta^h \tag{3.3}$$

$$\left[\frac{\partial W}{\partial p_j}\right] = -\sum_{h=1}^{H} \beta^h q_j^h \tag{3.4}$$

where $\beta^h = \frac{\partial W}{\partial u^h}\frac{\partial u^h}{\partial x}$ is the social marginal utility of income of the hth consumer or the welfare weight.

Differentiating (3.1) with respect to p_0 and p_j yields

$$\frac{\partial \bar{R}}{\partial p_0} = \sum_{i=1}^{n} \alpha_i p_i \frac{\partial Q_i}{\partial p_0} + H\bar{q}_0$$

and

$$\frac{\partial \bar{R}}{\partial p_j} = Q_j + \sum_{i=1}^{n} \alpha_i p_i \frac{\partial Q_i}{\partial p_j}$$

which on substituting into (3.2) and using (3.3) and (3.4) give the optimization conditions as

$$\left[\bar{q}_0 \sum_{h=1}^{H} \beta^h\right]\left[Q_j + \sum_{i=1}^{n} \alpha_i p_i \frac{\partial Q_i}{\partial p_j}\right] = \left[\sum_{h=1}^{H} \beta^h q_j^h\right]\left[H\bar{q}_0 + \sum_{i=1}^{n} \alpha_i p_i \frac{\partial Q_i}{\partial p_0}\right] \tag{3.5}$$

Let us assume that taxes on all non-rationed commodities are proportional to prices, i.e. $\alpha_j = \alpha$ for all j = 1,2,...,n, then (3.5) simplifies to

$$\left[\bar{q}_0 \sum_{h=1}^{H} \beta^h\right]\left[Q_j + \alpha \sum_{i=1}^{n} p_i \frac{\partial Q_i}{\partial p_j}\right] = \left[\sum_{h=1}^{H} \beta^h q_j^h\right]\left[H\bar{q}_0 + \alpha \sum_{i=1}^{n} p_i \frac{\partial Q_i}{\partial p_0}\right] \tag{3.6}$$

The budget constraint for all consumers may be written as

$$X = \sum_{i=1}^{n} p_i Q_i + H p_0 \bar{q}_0$$

where $X = \sum_{h=1}^{H} x^h$ is the total income of all consumers. Differentiating this equation with respect to p_0 and p_j yields

$$H\bar{q}_0 + \sum_{i=1}^{n} p_i \frac{\partial Q_i}{\partial p_0} = 0$$

and

$$Q_j + \sum_{i=1}^{n} p_i \frac{\partial Q_i}{\partial p_j} = 0$$

which on substituting into (3.6) leads to

$$(1-\alpha)\bar{q}_0 Q_j \left[\sum_{h=1}^{H} \beta^h\right] = H\bar{q}_0 (1-\alpha) \left[\sum_{h=1}^{H} \beta^h q_j^h\right] \tag{3.7}$$

When β^h are all not equal, (3.7) will hold true only if $\alpha = 1$, which means that all non-rationed commodities must be taxed at infinite rate.[2] In the case of a single person economy, the optimum tax policy implied zero taxes on all non-rationed commodities. Thus, the single person result does not generalize to the many person case.

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[2]This follows from the fact that $\alpha_i = \frac{t_i}{\bar{p}_i + t_i}$, $\bar{p}_i$ being the producer price so that $\alpha_i = 1.0$ will hold true only if t_i approaches infinity (in view of the fact that $\bar{p}_i \neq 0$ for all i).

The above conclusions may change, however if we do not impose the restriction that taxes on all non-rationed commodities are proportional to prices. In order to get stronger results it would be necessary to make some assumptions about the social welfare function. This is considered in the next section.

4. A SPECIFIC SOCIAL WELFARE FUNCTION

We assume that all consumers have identical tastes and differ only in income.[3] The preferences are given by the indirect utility function $u(\bar{q}_0, p_0, p, x)$. The government maximizes the social welfare function

$$W = \frac{1}{(1-\epsilon)} \int_a^{\infty} u^{1-\epsilon} f(x)dx \tag{4.1}$$

where a is the minimum income, f(x) the probability density function describing the distribution of total expenditure and ϵ is a measure of the government's aversion to inequality. If ϵ is zero, the social welfare is given by the mean utility level. As ϵ approaches infinity, social welfare is sensitive only to the utility of the poorest consumer in the economy. Note that Atkinson's (1970) welfare function is defined over money incomes whereas here we use the utility function (see Deaton 1977).

The government's revenue constraint as given in (3.1) can also be written as

$$\bar{R} = \sum_{i=1}^{n} t_i E(q_i) + t_0 \bar{q}_0 \tag{4.2}$$

[3]The differences due to size and composition of households to which consumers belong can be taken into account under this framework by means of consumer unit scales (see for instance Deaton and Muellbauer (1980), Muellbauer (1974, 1980), Kakwani (1977, 1980) and more recently Ray (1985).

where $E(q_i)$ is the mathematical expectation of q_i[4].

Using Roy's theorem, it can be shown that

$$\frac{\partial W}{\partial p_0} = -\bar{q}_0 E(u^{-\varepsilon}\lambda) \tag{4.3}$$

and

$$\frac{\partial W}{\partial p_j} = -E(u^{-\varepsilon} q_j \lambda) \tag{4.4}$$

where $\lambda = \frac{\partial u}{\partial x}$ is the marginal utility of income as defined in (2.3).

Utilizing Slutsky's equation it can be shown that

$$\frac{\partial \bar{R}}{\partial p_0} = \bar{q}_0[1-E(\phi)] \tag{4.5}$$

and

$$\frac{\partial \bar{R}}{\partial p_j} = E[q_j(1+\delta_j-\phi)] \tag{4.6}$$

where ϕ and δ_j are defined in (2.10) and (2.11), respectively.

Substituting (4.3), (4.4), (4.5) and (4.6) into the first-order conditions (3.2) yields

$$E[q_j(1+\delta_j-\phi)] = T_j E(q_j)[1-E(\phi)] \tag{4.7}$$

where

$$T_j = \frac{E(u^{-\varepsilon}\lambda q_j)}{E(u^{-\varepsilon}\lambda)E(q_j)}$$

It can be seen that when $\alpha_j = \alpha$ for all $j = 1, 2, \ldots, n$, (4.7) will hold true only if $\alpha_j = 1$ for all j, which confirms our earlier result that

[4]The expectation is evaluated over the probability distribution of the total consumer expenditure. This formulation uses the assumption that the demand function is the same for all consumers.

all non-rationed goods in the many person case must be taxed at infinite rates.

Let us write

$$E(q_j\delta_j) = E(q_jk_j) + \alpha_jE(q_j\eta^*_{jj}) \tag{4.8}$$

where $E(q_jk_j)$ is the average compensated change in the demand for good j as a result of imposing taxes on goods other than j. (4.8), thus gives us a measure of the substitutability of good j. Substituting (4.8) into (4.7) gives

$$\alpha_j = \frac{1}{E(q_j\eta^*_{jj})}\left[(1-T_j)E(q_j)[1-E(\phi)] + [E(q_j)E(\phi) - E(q_j\phi)] + E(q_jk_j)\right] \tag{4.9}$$

Since λ is the marginal utility of income, it must be a decreasing function of income and $\varepsilon > 0$ implies that $u^{-\varepsilon}$ decreases with x, so that $E(u^{-\varepsilon}\lambda)$ and q_j will be negatively correlated which would mean

$$E(u^{-\varepsilon}\lambda q_j) < E(u^{-\varepsilon}\lambda)E(q_j)$$

which implies that $T_j < 1$ for all j. Also as ε increases, the magnitude of correlation between $u^{-\varepsilon}\lambda$ and q_j increases, so that $(1-T_j)$ will be an increasing function of ε. Since the first term in the righthand side of (4.9) is positive, α_j will be an increasing function of ε, i.e. the more a society is concerned about inequality, the greater will be the tax on the non-rationed commodity.

Further, if commodity j is an absolutely inelastic commodity in the sense that its consumption is completely insensitive to changes in income, then $T_j = 1$ and $E(q_j Q) = E(q_j)E(Q)$ which imply that the first two terms in the righthand side of (4.9) will be zero. This means that the tax on commodity j will depend on the third term. If in addition commodity j is of limited substitutability, it will attract zero tax rate.

The substitution term $E(q_j k_j)$ is likely to be positive for necessities and negative for luxuries and, therefore, this term will dictate that necessities should attract higher tax rates. But this effect can be offset by the first term which implies lower tax rates on necessities. If the social welfare function is highly egalitarian, i.e. ϵ is high, the first term in (4.9) will dominate the third term. The policy implication of this is that the government will tax luxury goods more heavily than necessities. Thus, the major conclusion emerging from this section is that tax rates on non-rationed commodities should not be uniformly proportional to prices. If, however, the restriction of uniformity is imposed, then the optimum policy will be to tax all non-rationed commodities at infinite rate which in our view is not a feasible solution.

5. LINEAR EXPENDITURE SYSTEM

The demand functions under rationing for the linear expenditure system is given by (Neary and Roberts 1980):

$$q_i = \gamma_i + \frac{\beta_i}{(1-\beta_0)p_i}\left[x - \sum_{i=1}^{n} p_i \gamma_i - p_0 \bar{q}_0\right] \tag{5.1}$$

where γ_i is the subsistence consumption of the ith commodity and β_i is its marginal budget share so that

$$\sum_{i=1}^{n} \beta_i = (1-\beta_0) \ . \tag{5.2}$$

Using the Slutsky equation, immediately yields the compensated price elasticities as

$$\eta^*_{ji} = \frac{\beta_i(q_j-\gamma_j)}{(1-\beta_0)q_j} \ , \qquad i \neq j$$

$$= \frac{\beta_i(q_j-\gamma_j)}{(1-\beta_0)q_j} - \frac{(q_j-\gamma_j)}{q_j} \ , \qquad i = j$$

which gives

$$\delta_j = \frac{(q_j-\gamma_j)}{q_j}(\phi-\alpha_j) \tag{5.3}$$

where $\phi = \sum_{i=1}^{n} \frac{\beta_i}{(1-\beta_0)} \alpha_i$.

When the Engel curves are linear, ϕ in (2.10) will be non-stochastic which simplifies (4.7) to

$$E(q_j\delta_j) = -E(q_j)\ (1-\phi)(1-T_j)$$

which in view of (5.3) becomes

$$\alpha_j = \phi + \frac{E(q_j)(1-\phi)(1-T_j)}{E(q_j-\gamma_j)} \tag{5.4}$$

Substituting the demand function (5.1) into (5.4) leads to

$$\alpha_j = \phi + \frac{(1-\phi)E[u^{-\varepsilon}\lambda(\mu-x)]}{E(u^{-\varepsilon}\lambda)(\mu-\sum_{i=1}^{n} p_i\gamma_i - po\bar{q}o)}$$

where $\mu = E(x)$, the mean total expenditure. This equation demonstrates that θ_j is same for all j, i.e. the taxes on non-rationed commodities must be uniformly proportional. Thus, it follows from the previous sections that the non-rationed commodities must be taxed at infinite rate. This is an important finding. It demonstrates that the linear expenditure system cannot be used to find a sensible optimal commodity tax structure under rationing.

6. AN ALTERNATIVE MODEL OF RATIONING

We now consider an alternative model of rationing in which the government provides a fixed amount of one commodity, say, food at a subsidized price. Suppose there are n commodities $q_1, q_2, \ldots, q_n$ in the economy and $\bar{q}_1$ is the amount of the first commodity provided at a subsidized price $\bar{p}_1$. If $p = (p_1, p_2, \ldots, p_n)$ is the vector of market prices, the consumer maximizes his or her utility function

$$u = u(q_1, q_2, \ldots, q_n) \tag{6.1}$$

subject to the budget constraint

$$x = \bar{p}_1 \bar{q}_1 + (q_1 - \bar{q}_1)p_1 + p_2 q_2 + \ldots p_n q_n \tag{6.2}$$

Note that if $q_1 > \bar{q}_1$, the consumer satisfies his or her excess demand of the first commodity over $\bar{q}_1$ by buying it at the prevailing market price. Thus, there is a dual market for the first commodity.

It can be seen that the above model is equivalent to the usual consumer demand model when the consumer has been given a cash subsidy of

$(p_i-\bar{p}_i)\bar{q}_i$.[5] Thus, the demand equation of the ith commodity will be given by

$$q_i = q_i(p, x + (p_i-\bar{p}_i)\bar{q}_i) \tag{6.3}$$

Note that if $\bar{p}_i$ is fixed, this demand equation is not a homogenous function of degree zero in prices p and income x.

Substituting (6.3) into (6.1) yields the indirect utility function of the consumer with income x as

$$u = u[p, x + (p_i-\bar{p}_i)\bar{q}_i] \tag{6.4}$$

which on differentiating with respect to p_i and p_j and using the Roy's identity

$$\frac{\partial u}{\partial p_i} = -\lambda(q_i-\bar{q}_i) \tag{6.5}$$

and

$$\frac{\partial u}{\partial p_j} = -\lambda q_j \tag{6.6}$$

respectively.

Note that $\frac{\partial u}{\partial p_i} = 0$ if $q_i = \bar{q}_i$ which means that the consumers who do not buy the rationed commodity at the market price are not adversely affected by the increase in the market price of that good.

The optimal problem now involves determining tax rates $t_1, t_2, \ldots, t_n$ so that consumer's indirect utility function (6.4) is maximized subject to the government's revenue constraint

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[5]It is assumed that the ration quota is always less than the actual requirement of that commodity.

$$R = \sum_{i=1}^{n} t_i q_i - (p_1 - \bar{p}_1)\bar{q}_1 \qquad (6.7)$$

which is based on the assumption that the government buys the rationed commodity at the producer's price and sells it to the consumer at a lower price of $\bar{p}_i$.[6]

Using the Slutsky equation and the demand equation (6.3), it can be shown that

$$\frac{\partial R}{\partial p_1} = q_1(1+\delta_1-\phi) - \bar{q}_1 \qquad (6.8)$$

and

$$\frac{\partial R}{\partial p_j} = q_j(1+\delta_j-\phi) \qquad (6.9)$$

where ϕ_j and δ_j are defined in (2.10) and (2.11), respectively.

Substituting (6.5), (6.6), (6.8) and (6.9) into the first order optimization condition for a single person we obtain

$$(q_1-\bar{q}_1)\delta_j = -\phi\bar{q}_1 + q_1\delta_1 \qquad (6.10)$$

Suppose that all taxes are uniformly proportional, i.e. $\alpha_i = \alpha$ for all i which will imply δ_1 and δ_j equal to zero for all j = 2,...,n. Under these conditions (6.10) will be satisfied only if $\bar{q}_1 = 0$, i.e. when no commodity is subsidized. Thus, when a fixed quantity of any commodity is provided at a subsidized price, the optimum tax policy dictates that we do not have all uniformly proportional taxes.

[6]Note that the government collects revenue on the first commodity only on the amount bought in the open market, i.e. $(q_1-\bar{q}_1)$.

Equation (6.10) can also be written as

$$\alpha_j = \frac{-\phi\bar{q}_1 + q_1k_1 + \alpha_1q_1\eta^*_{11} - (q_1-\bar{q}_1)\ k_j}{(q_1-\bar{q}_1)\eta^*_{jj}} \tag{6.11}$$

where k_j is the compensated proportional change in the demand for the jth good as a result of imposing taxes on goods other than j. k_j is likely to be positive for necessities and negative luxury goods. It can be seen from (6.11) that α_j will be higher if the jth commodity is a necessity. Further, the magnitude of α_j is inversely proportional to the compensated own price elasticity. However, the most interesting finding is that α_j is a decreasing function of k_1, an implication of which is that if the rationed commodity is a necessity, the taxes on non-rationed item is generally a necessity and therefore, should attract greater tax than the non-rationed item.

7. CONCLUSIONS

In this paper we have investigated the implications of the optimal commodity taxation in the presence of rationing. The analytical results presented in the paper raise the fundamental issue of the relevance and usefulness of the standard optimal commodity tax results in evaluating existing indirect taxes and providing policy advice. Following are some of the implications for tax policies that emerge from this analysis.

1. In the presence of a single person economy the optimal policy dictates that the rationed commodity bears the entire tax. In the context of developing countries, this result implies that if the government has a fixed budget to subsidize certain commodities, the optimal policy will

be to subsidize only the rationed commodities (non-rationed commodities should receive zero subsidy).

2. The single person result of zero tax on non-rationed commodities does not generalize to the many person case. In the many person case if we impose the restriction that taxes on all non-rationed commodities are proportional to prices, the optimal policy will tax all non-rationed commodities at infinite rate.
3. The linear expenditure system always results in uniformly proportional taxes on non-rationed commodities. It follows that the non-rationed commodities must always be taxed at infinite rate. Thus, the LES cannot be used to find a sensible optimal commodity tax structure under rationing. The linearity/separability assumptions of LES are already known to have some restrictive implications for optimal tax theory in the standard formulation. We have extended them to the rationed case.
4. The tax on a non-rationed commodity should be an increasing function of the inequality aversion parameter, i.e. the more a society is concerned about inequality, the greater will be the tax on the non-rationed commodity.
5. When a fixed quantity of any commodity is provided at a subsidized price, the optimal tax policy dictates that we do not have all uniformly proportional taxes. This result differs from the standard optimal commodity tax result which states that all taxes should be uniformly proportional for the single consumer economy.

A useful extension of this study and one that we hope to carry out in further work would be the empirical calculation of optimal commodity taxes in the presence of rationing and within the framework of a nonlinear (and, hence, more realistic) demand model. It is worth noting that while a good deal can be said a priori about optimal commodity taxes if preferences are separable, very little can be asserted if they are not. As Ray (1986)'s results have shown, optimal commodity taxes under realistic demand systems bear very little resemblance to LES-based tax rates, especially from the viewpoint of an equity conscious planner. As we have shown in this paper, the situation gets more complicated even for the restrictive rationed LES. Empirical calculation of optimal commodity taxes under rationed nonlinear demand systems, while requiring complex and expensive estimation proceedings, would be a valuable contribution to the optimal tax literature, especially in developing countries.

REFERENCES

Ahmad, E. and N.H. Stern (1984), "The theory of Reform and Indian Indirect Taxes", *Journal of Public Economics*, 25, 259-298.

Atkinson, A.B. (1970), "On the Measurement of Inequality", *Journal of Economic Theory*, 2, 244-63.

Atkinson, A.B. and J.E. Stiglitz (1972), "The structure of indirect taxation and economic efficient", *Journal of Public Economics*, 1, 97-119.

Atkinson, A.B. and J.E. Stiglitz (1980), *Lectures on Public Economics* (McGraw-Hill, New York).

Deaton, A.S. (1974), "A reconsideration of the empirical implications of additive preferences", *Economic Journal*, 84, 338-348.

Deaton, A.S. (1977), "Equity, efficiency and the structure of indirect taxation", *Journal of Public Economics*, 8, 299-312.

Deaton, A.S. and Muellbauer, J. (1980), *Economics and Consumer Behaviour, Cambridge University Press.*

Deaton, A.S.and N.H. Stern (1986), "Optimally uniform commodity taxes, taste differences and lump-sum grants", *Economics Letters*, 20, 263-266.

Diamond, P.A. and J.A. Mirrlees (1971), "Optimal Taxation and Public Production II: Tax Rules", *American Economic Review*, 61, 261-278.

Harris, R.G. and J.G. Mackinnon (1978), "A single technique for computing optimal tax equilibria", *Journal of Public Economics*, 11, 197-212.

Howard, D.H. (1977), "Rationing, quantity constraints and consumption theory", *Econometrica*, 45, 399-412.

Kakwani, N. (1977), "On the Estimation of Consumer-Unit Scale", *Review of Economics and Statistics*, 59, 507-10.

Kakwani, N. (1980), *Income Inequality and Poverty: Methods of Estimation and Policy Applications*, Oxford University Press, New York.

Muellbauer, J. (1974), "Prices and Inequality: the U.K. Experience", *Economic Journal*, 84, 32-55.

Neary, J.P. and K.W.S. Roberts (1980), "The theory of household behaviour under rationing", *European Economic Review*, 13, 25-42.

Newbery, D. and N.H. Stern (eds.) (1987), *The Theory of Taxation for Developing Countries* (OUP and World Bank).

Pollak, R.A. (1969), "Conditional demand functions and consumption theory", *Quarterly Journal of Economics*, 83, 60-78.

Ramsey, F.P. (1927), "A contribution to the theory of taxation", *Economic Journal*, 37, 47-61.

Ray, R. (1985a), "A dynamic analysis of expenditure patterns in rural India", Journal of Development Economics, 19, 283-297.

Ray, R. (1985b), "Prices, Children and Inequality: Further Evidence for the United Kingdom, 1965-82", The Economic Journal, 95, 1069-1077.

Ray, R. (1986), "Sensitivity of 'optimal' commodity taxes to alternative demand functional forms: an econometric case study of India", Journal of Public Economics, 31, 253-268.

Rothbert, E. (1941), "The measurement of change in real income under conditions of rationing", Review of Economic Studies, 8, 100-107.

Sah, R.K. (1987), "Queues, Rations and Market: comparison of outcomes for the poor and the rich", American Economic Review, 77(1), 69-77.

Tanzi, V. (1987), "Quantitative Characteristics of the Tax Systems of Developing Countries", in Newbery, D.M.G. and N.H. Stern (eds.), 1987.

Tobin, J. (1952), "A survey of the theory of rationing", Econometrica, 20, 512-553.

Tobin, J. and H.S. Houthakker (1951), "The effects of rationing on demand elasticities", Review of Economic Studies, 18, 140-153.

Weitzman, M.L. (1977), "Is the price system or rationing more effective in getting a commodity to those who need it most?", Bell Journal of Economics, 8, 517-524.

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